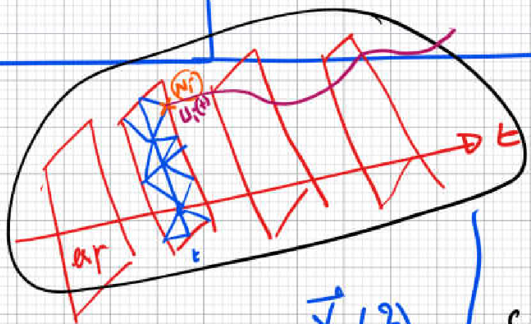
$$\frac{\partial u}{\partial t}(x,y,t) - \Delta u(x,y,t) = f(x,y,t)$$



M_1

M_2

Soit $0<t<T$

$$\left[\frac{\partial u}{\partial t}(x,y,t) - \Delta u(x,y,t) = f(x,y,t)\right] v$$

$$\int_\Omega \frac{\partial u}{\partial t}(x,y,t)\, v - \int_\Omega \Delta u(x,y,t)\, v = \int_\Omega f(x,y,t)\, v$$

$\left\{ \cancel{v(x,t)}, \cancel{v(x)} \right.$ $\qquad$ || Green

$$\frac{\partial}{\partial t}\int_\Omega u(x,y,t)\, v + \int_\Omega \nabla u(x,y,t)\cdot\nabla v(x,y) = \int_\Omega f(x,y,t)\, v$$

$u_h(x,y,t), v(x,y) \in V_h = \mathrm{Vect}(\varphi_1, \dots, \varphi_N)$ nombre de noeuds % à l'espace

Choix: (1) $u_h(x,y,t) = \sum_{i=1}^{N} u(N_i;t)\,\varphi_i(x,y)$ *

(2) $v_h = \varphi_j \quad 1\le j\le N$

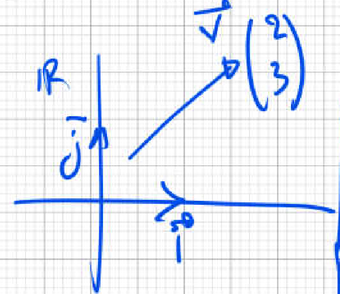
$$\frac{\partial}{\partial t}\int_\Omega\left(\sum_{i=1}^N u_i(t)\varphi_i(\vec x)\right)\varphi_j(\vec x) + \int_\Omega \nabla\left(\sum_{i=1}^N u_i(t)\varphi_i(\vec x)\right)\cdot\nabla(\varphi_j(\vec x)) = \int_\Omega f(\vec x,t)\,\varphi_j(\vec x) \quad 1\le j\le N$$

$$\sum_{i=1}^N\left\{\left[\int_\Omega \varphi_i(\vec x)\varphi_j(\vec x)\right]\frac{d}{dt}u_i(t)\right\} + \sum_{i=1}^N u_i(t)\left\{\int_\Omega \nabla\varphi_i\cdot\nabla\varphi_j\right\} = \int_\Omega f(\vec x,t)\,\varphi_j(\vec x) \quad 1\le j\le N$$

Posons, $M = \left(\int_\Omega \varphi_i\varphi_j\right)$; $K = \left(\int_\Omega \nabla\varphi_i\cdot\nabla\varphi_j\right)$; $F(t) = \left[\int f\varphi_j\right]$

et $U(t) = \begin{bmatrix} u_1(t) \\ \vdots \\ u_N(t) \end{bmatrix}$

$$\boxed{M\frac{dU}{dt} + KU = F}$$



$0 \quad t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5 \quad t_6 \quad T$

$t_n \;/\; 0\le n\le 7$

Soit $0\le n\le 7$

$$\frac{\partial u}{\partial t}(x,y,t_n) - \Delta u(x,y,t_n) = f(x,y,t_n)$$

Notation : $u(x,y,t) = U^n(x,y)$; $u(x,y,t_n) = u_n(x,y)$

$\frac{\partial u}{\partial t}(x,y,t_n)$ explicite : $\frac{\partial u}{\partial t}(x,y,t_n) \simeq \frac{u^{n+1}(x,y) - u^n(x,y)}{\Delta t}$

implicite : $\frac{\partial u}{\partial t}(x,y,t_n) \simeq \frac{u^n(x,y) - u^{n-1}(x,y)}{\Delta t}$

$\Delta t = t_{n+1} - t_n$

$$\frac{u^{n+1} - u^n}{\Delta t} - \Delta u^n = f^n$$

$$u^{n+1} = u^n + (\Delta t)\,\Delta u^n + (\Delta t)\, f^n$$

$$\int_\Omega u^{n+1} v = \int_\Omega u^n v + \Delta t\int_\Omega \Delta u^n v + \Delta t\int_\Omega f^n v \qquad v_{|\partial\Omega} = 0$$

$$\int_\Omega u^{n+1} v = \int_\Omega u^n v - (\Delta t)\int_\Omega \nabla u^n\cdot\nabla v + \Delta t\int_\Omega f^n v$$

Choix. $u^n = \sum_{i=1}^N u_i^n\varphi_i$ et $v = \varphi_j \quad 1\le j\le N$

$$\sum_{i=1}^N u_i^{n+1}\int_\Omega \varphi_i\varphi_j = \sum_{i=1}^N u_i^n\int_\Omega\varphi_i\varphi_j - (\Delta t)\sum_{i=1}^N u_i^n\int_\Omega \nabla\varphi_i\cdot\nabla\varphi_j + (\Delta t)\int_\Omega f^n\varphi_j \quad 1\le j\le N$$

$$MU^{n+1} = MU^n - (\Delta t)KU^n + \Delta t\, F^n$$

$$U^{n+1} = M^{-1}\left[M - \Delta t K\right]U^n + \Delta t\, M^{-1}F^n$$

$U^0 =$